"Let no one say that I have said nothing new…the arrangement of the subject is new. When we play tennis, we both play with the same ball, but one of us places it better."

-Blaise Pascal

ACKNOWLEDGEMENT

This book is dedicated to my 8th grade math teacher, Stan Walters, for "raising the bar" and for recognizing my mathematical ability.

I also thank all the teachers, parents and students who have learned from me and allowed me to learn from them.

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PREFACE

This book did not start out as a book. Starting January 1, 2014 the new 2014 exam replaced the old GED test and the Algebra bar was raised. There is now more Algebra! Much more Algebra!!! My original plan was to help teachers present the new math. I went through the new pre-test, practice test, and Readiness Test and developed an outline for professional development workshops for teachers. The outline became the "Sweet 16" chapters in this book. Hopefully, *HSE ALGEBRA SWEET 16* will reach many teachers and students and simplify the Algebra concepts required to pass the new exam.

HSE ALGEBRA SWEET 16

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4
Functions

A RELATION is a set of ordered pairs (points).

Example: A is a relation.

$A = \{(-1,2), (1,2), (2,6), (4,3)\}$

The Domain is the set of all the x-coordinates (the first number in each pair): {-1, 1, 2, 4}.
The Range is the set of all the y-coordinates (the second number in each pair): {2, 6, 3}.
It is not necessary to repeat the coordinate 2 in the Range.

A relation can also be written in an xy table. The relation above is represented below.

x	y
-1	2
1	2
2	6
4	3

A FUNCTION is a special relation where every x matches up with only one y.

The relation above, $\{(-1,2), (1,2), (2,6), (4,3)\}$, is a function.
It's a function because every x has only one y.

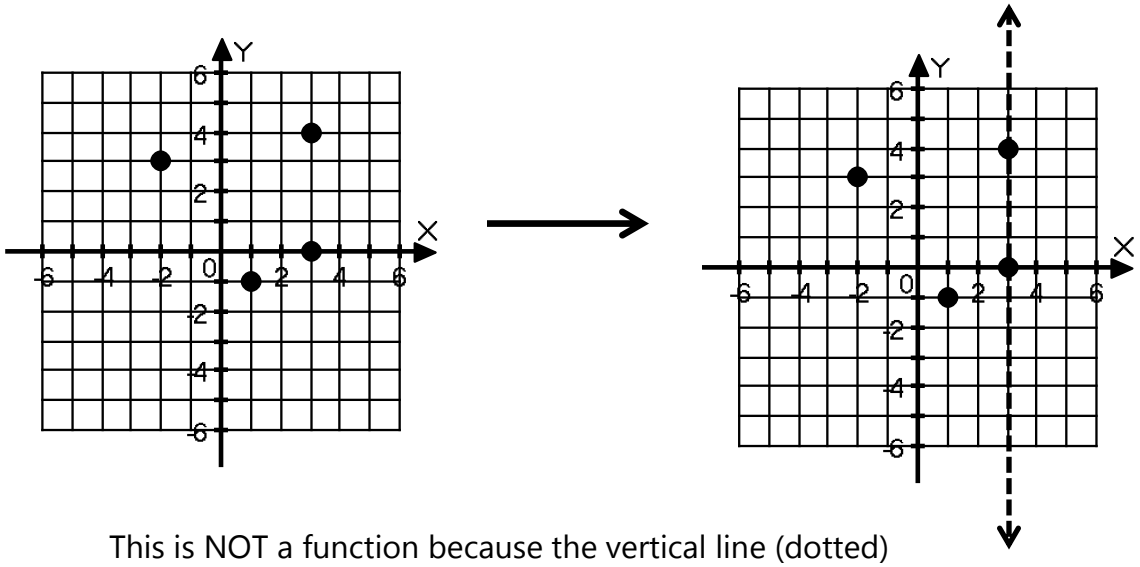
The relation below is NOT a function.
The x-coordinate 2 matches up with two different y's (5 and 7).

x	y
1	3
2 →	5
2 →	7

The Vertical Line Test:
There is a visual way to quickly tell if a relation is a function. If the relation is graphed, we can check to see if any two points are vertically in line with each other. If you can draw a vertical line through two points, then the relation is NOT a function because those two points will have the same x and two different y's.

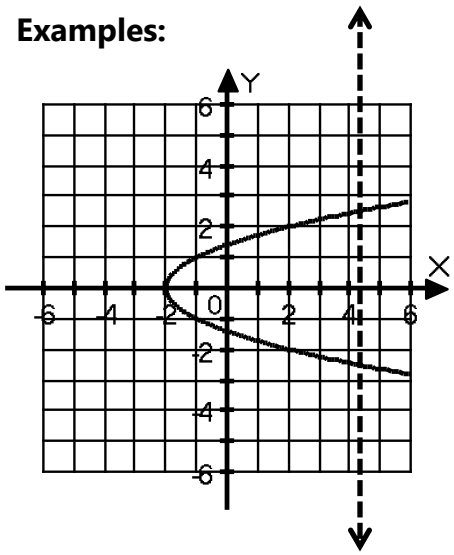
Example: $\{(-2,3),(3,4),(1,-1),(3,0)\}$

x	*y*
-2	3
3 →	4
1	-1
3 →	0

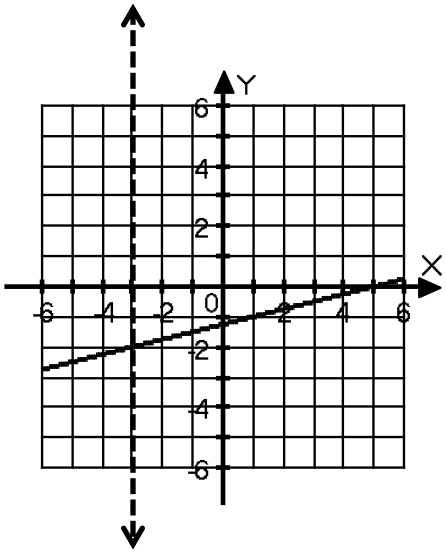


This is NOT a function because the vertical line (dotted) goes through two points. Looking closer, you can see that both these points have the same x (3) and different y's (4 and 0).

Examples:



This is NOT a function.
You can draw several vertical lines that intersect two points.

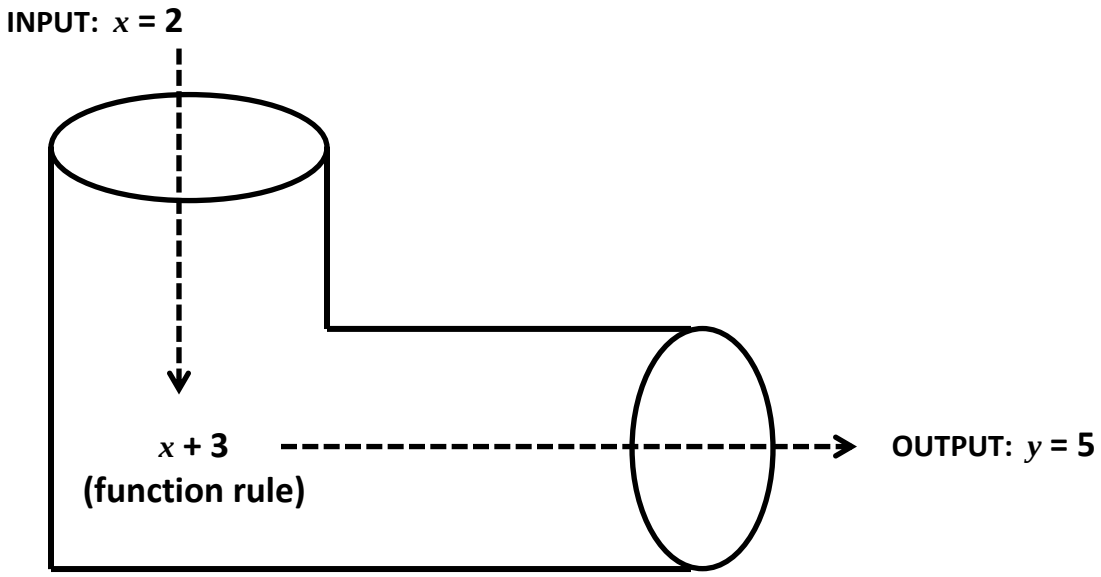


This is a function.
You cannot draw a vertical line that intersects two points.

Function Patterns:

Imagine a machine that eats numbers and spits them out! That's what a function does.

The number 2 enters the machine and the number 5 comes out. Then 6 goes in and 9 comes out. Then 12 goes in and 15 comes out.



The number going into the machine is called the input and labeled with the letter x.
The number coming out of the machine is called the output and labeled with the letter y.

As you can see, 3 is added to each input to get the output.
So, the function rule for this function is $x + 3$. Every time x goes in we add 3 to it.

Here is the table for the above function.

x	y
2	5
6	9
12	15

The table can also be written horizontally.

x	2	6	12
y	5	9	15

This function can also be written as an equation:

$y = x + 3$

There is another function notation you need to know. You will probably see it on the HSE Exam.

Remember that y is the output. y is interchangeable with the symbol $\boldsymbol{f(x)}$.

$f(x)$ is read "the function of x".

$\boldsymbol{y = x + 3}$ and $\boldsymbol{f(x) = x + 3}$ mean the same thing.

$f(x)$ is the output.

If you are asked to find $f(2)$, you are being asked to find the output when 2 is input.
Therefore, you input 2 into the function rule and output the number 5.
Therefore, $f(2) = 5$ when the rule is $x + 3$.

Example:

$f(x) = 5x$
Find $f(9)$.

Answer: find $f(9)$ means $x = 9$. So plug 9 into the function rule $5x$.
$f(9) = 5(9) =$ **45**.

Example:

Study the table. Figure out what the function rule is to finish the table.

$\boldsymbol{x}$	$\boldsymbol{f(x)}$
0	-2
3	1
6	4
14	

Answer: The output $f(x)$ is always 2 less than x. Therefore, the rule is $x - 2$.
$f(14) =$ **12**.

Example:

$f(x) = 3x - 4$
Find $f(12)$.

Answer: $f(12) = 3(12) - 4 =$ **32**.

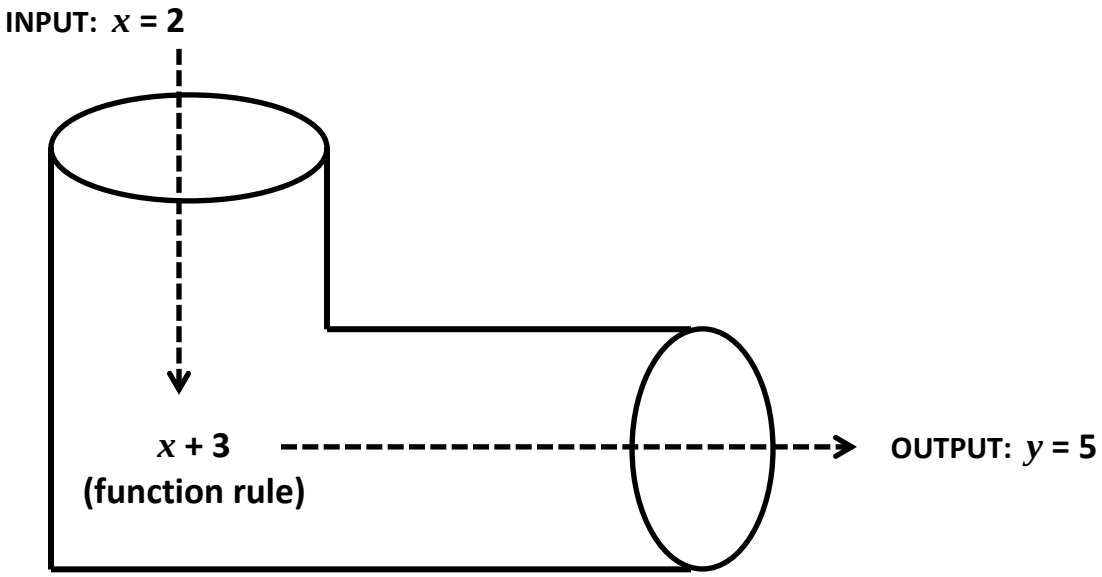
Example:

$f(x) = x^2 - 8$
Find $f(5)$.

Answer: $f(5) = 5^2 - 8 = 25 - 8 =$ **17**.

Inverse Function:

An inverse function just works in reverse. Start with the output and ask yourself, what would you do to get back to the input?



Looking at this function above, the inverse function would take the number 5 and subtract 3 to get the original input of 2. Therefore, the inverse function would be $x - 3$.

The inverse function notation is $f^{-1}(x)$.

NOTE: You would think you would use notation starting with the y and work back to x. NOT SO! We always start with x and go toward y even when we are writing the inverse function.

Example:

If the function is $f(x) = x + 3$, then the inverse function is $f^{-1}(x) = x - 3$.

If the function says to multiply by 3 and then add 5, the inverse function would be to subtract 5 and then divide by 3.

Example:

$f(x) = 3x + 5$.
Find the inverse function $f^{-1}(x)$.

Answer: $f^{-1}(x) = \dfrac{x-5}{3}$

Chapter 4 Practice:
Functions

Given the relation {(3,5), (2,6), (0,8), (2,3)}, answer the following:

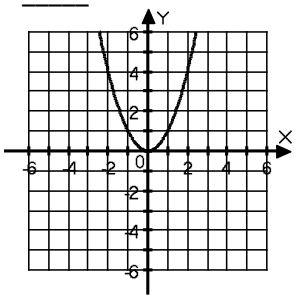
1) _____ What is the domain?

2) _____ What is the range?

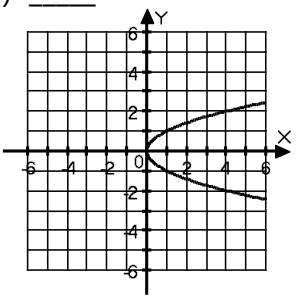
3) _____ Is the relation a function?

Are the following functions?

4) _____



5) _____



Finish the tables.

6)

x	y
2	7
4	9
10	
12	

7)

x	y
-1	-3
2	6
4	
5	

Find $f(x)$.

8) _____ $f(x) = x + 11$
$f(5) = ?$

9) _____ $f(x) = 2x^2 + 3x - 1$
$f(10) = ?$

Which is the inverse function of $f(x) = 2x - 5$?

10) _____

A $5 + 2x$

B $\frac{x}{2} + 5$

C $\frac{x+5}{2}$

Chapter 4 KEY:
Functions

Given the relation {(3,5), (2,6), (0,8), (2,3)}, answer the following:

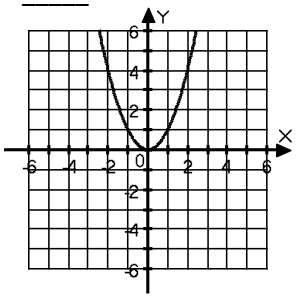
1) $\{3,2,0\}$ What is the domain?

2) $\{5,6,8,3\}$ What is the range?

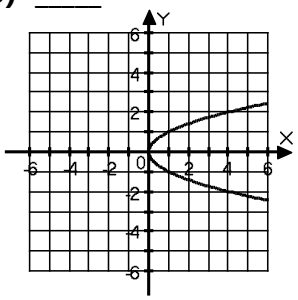
3) *no* Is the relation a function?

Are the following functions?

4) *yes*



5) *no*



Finish the tables.

6)

x	y
2	7
4	9
10	**15**
12	**17**

***f*(*x*) = *x*+5**

7)

x	y
-1	-3
2	6
4	**12**
5	**15**

f*(*x*) = 3*x

Find $f(x)$.

8) 16 $f(x) = x + 11$

$f(5) = 5 + 11$

9) 229 $f(x) = 2x^2 + 3x - 1$

$f(10) = 2(10)^2 + 3(10) - 1$

Which is the inverse function of $f(x) = 2x - 5$?

10) C

A $5 + 2x$

B $\frac{x}{2} + 5$

C $\frac{x+5}{2}$