

# ALGEBRA HIGHLIGHTS

**MADE SIMPLE**

---

---

The ABC'S OF ALGEBRA  
The ABC'S OF ALGEBRA  
RESOURCE MANUAL  
RESOURCE MANUAL

---

---

By  
Jay Snyder  
[*Jmath.net*]

*“Education is the best provision for the journey to old age.”*

*-Aristotle*

## ACKNOWLEDGEMENT

*This book is dedicated to my mother for her love,  
unselfishness and encouragement.*

*I also thank all the teachers, parents and students who have  
learned from me and allowed me to learn from them.*

*And a special thank you to Byron Zachary and Robert Saldivar.  
Their friendship and support made this publication possible.*

# CONTENTS

Acknowledgement .....	ii
Contents .....	iii
Preface .....	iv
Why x? .....	v
1 Types of Numbers .....	1
2 Sets & Subsets .....	6
3 Properties .....	13
4 Negative Numbers .....	23
5 Absolute Value .....	30
6 Variables & The Polynomial .....	33
7 Exponents .....	38
8 Simplifying Expressions.....	45
9 Evaluating Expressions .....	52
10 Multiplying Polynomials .....	55
11 Factoring .....	61
12 Solving Equations – a matter of balance .....	74
13 Solving Proportions – the CD way .....	83
14 Solving Equations By Factoring .....	91
15 Inequalities .....	101
16 Graphing .....	109
17 Word Problems.....	147
18 Solving Systems .....	165
19 Rational Expressions .....	181
20 Irrational Expressions .....	194
21 Pythagoras & The Distance Formula .....	211
22 The Quadratic Formula .....	222
23 Trigonometry .....	228
Practice for Mastery Answer Keys .....	237
Index .....	250

## PREFACE

*Algebra Highlights Made Simple* is my attempt at clarifying in an organized, progressive manner the concepts taught in algebra. It is my hope that this book will be easy to follow and understand. The written explanations and illustrations are my thoughts that have been put to paper after 36 years of teaching.

Every teacher tweaks, changes, edits, and “fine tunes” their presentations over the years. This book is me. It is my presentation. My goal: keep it simple and ultimately make algebra easier to understand.

$$\{3, 4, 5\} \cup \{2, 4, 6\} = \{2, 3, 4, 5, 6\}$$

## 2 SETS & SUBSETS

A set is a collection of numbers or objects. These numbers or objects are called the elements or members of the set. Members of a set are grouped together in braces or brackets { }.

$\in$  is the symbol that means “is an element of.”  
 $\notin$  means “is not an element of.”

$2 \in \{1, 2, 3\}$   
2 is an element of the set 1, 2, 3.  
 $4 \notin \{1, 3, 5\}$   
4 is not an element of the set 1, 3, 5.

Sets can be named 3 ways:

- 1) by **roster**  
listing all the elements of the set
- 2) by **rule**  
writing a description
- 3) by **graph**  
points graphed showing a picture  
of the set

Example 1:

Specify the letters in the word “bigger” as a set by roster.

In other words, list the letters.

Answer: {b, e, g, r}

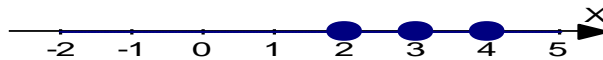
Example 2:

Specify the set  $\{a,b,c\}$  by rule.  
In other words, describe the set.

Answer: {the first 3 letters of the alphabet}

Example 3:

Specify the set of whole numbers between 1 and 5 with a graph.



Empty Sets:

A set can have no members.

A set with no members is called an *empty set* or *null set* and can be written as  $\phi$  or  $\{\}$ .

Infinite Sets:

$E = \{5,6,7,\dots\}$  the 3 dots are read “and so on.”

E is a set containing the elements 5,6,7 “and so on.”

E is an *infinite set* because it has a never-ending set of numbers.

Finite Sets:

$G = \{x,y,z\}$

G is a *finite set* because it has a definite number of elements. Set G has exactly 3 elements and therefore is finite. The empty set is also considered a finite set.

Equal Sets:

Two sets that have the exact same elements are called *equal sets*, regardless of the order that they are listed.

$$A = \{1,2,3\} \qquad B = \{3,2,1\}$$

Sets A & B are equal sets.

## The Union of Two Sets:

The math symbol for union is  $\cup$ .

*Union* means to unite. Finding the union of two sets is to unite the members from both sets into 1 set.

Example:

$$A = \{1,2,3,4\}$$

$$B = \{3,4,5,6\}$$

$$A \cup B = \{1,2,3,4,5,6\}$$

NOTE: It is not necessary to list the 3 and 4 twice.

## The Intersection of Two Sets:

The math symbol for intersection is  $\cap$ .

*Intersection* means to list all the members the sets have in common in 1 set.

Example:

$$N = \{\text{red, white, blue}\}$$

$$Q = \{\text{orange, blue, green}\}$$

$$N \cap Q = \{\text{blue}\}$$

## Disjoint Sets:

Two sets that have no elements in common are called *disjoint sets*.

Example:

$$R = \{\text{odd counting numbers}\}$$

$$S = \{\text{even counting numbers}\}$$

R and S are disjoint sets. They have no elements in common.

## Subsets:

The math symbol for subset is  $\subset$ .

Any part of or all of a set is a *subset* of that set.

Example:

$$A = \{1\}$$

$$B = \{1,2\}$$

$$C = \{1,2,3\}$$

$$A \subset C \quad (\text{A is a subset of C})$$

A is a subset of C because 1 is part of the set 1,2,3.

B is a subset of C because 1 & 2 are a part of the set 1,2,3.



C is a subset of C because 1,2 & 3 are a part of the set 1,2,3.  
(Any set is always a subset of itself)  
NOTE: the empty set ( $\phi$ ) is a subset of all sets.

Example:

List all of the sets that are subsets of {2,4,6}.

Answer:  $\phi$ , {2}, {4}, {6}, {2,4}, {2,6}, {4,6}, {2,4,6}

There are 8 subsets of the set {2,4,6}.

To find how many subsets a set has:

- 1) Count the number of elements in the set.
- 2) Find  $2^n$  where  $n$  is the number of elements in the set.
- 3)  $2^n$  is the number of subsets.

The previous example has 3 elements in the set. Therefore, it has  $2^3$  or 8 subsets.

Closure:

Finding if a set has *closure* under a given operation depends upon *both* the operation and the set of numbers involved.

A set is "closed" if you can take any 2 numbers in that set, perform the operation stated and always get another number in that same set.

Example 1:

Given the set  $A = \{1,2,3\}$

Is A closed under the operation of addition?

Answer: No, because  $2 + 3 = 5$ .  
2 & 3 are both members in the given set, but the result 5 is not in that set.

Example 2:

Given  $B = \{\text{whole numbers}\}$

Is  $B$  closed under multiplication?

Answer: Yes. Pick any 2 whole numbers and multiply them together. You will always get another whole number. Therefore, the set of whole numbers is closed under multiplication.

I think of the exclusive clubhouse membership. The members of the set are the members of the clubhouse. No new members are allowed. The membership is closed. If you perform an operation and the result is someone who is not already a member of the exclusive club, then the set is not closed under that operation. To keep closure, the result must already be a member of the club or set.



# PRACTICE FOR MASTERY

## chapter 2

I. Name the following sets by roster.

- \_\_\_\_\_ {whole numbers less than 4}
- \_\_\_\_\_ {last 3 letters of the alphabet}
- \_\_\_\_\_ {colors that make up the American flag}

II. Name the following sets by rule.

- \_\_\_\_\_ {1,2,3,4}
- \_\_\_\_\_ {b,l,u,e}
- \_\_\_\_\_ {1,3,5,7}

III. Graph the following sets.

- {all real numbers greater than 2}



- {-2, 3, 3/2}



- {all real numbers less than or equal to -1}



- {all integers between -2 and 3}



5. {all real numbers  $x$  such that  $-1 \leq x < 1$ }



IV. Name the Union or Intersection of the sets given.

$$A = \{2, 4, 6\}$$

$$S = \{\text{odd integers}\}$$

$$M = \{\text{even counting numbers}\}$$

$$B = \{3, 4, 5\}$$

$$T = \{\text{counting numbers}\}$$

$$N = \{\text{odd counting numbers}\}$$

1.  $A \cup B$  \_\_\_\_\_
2.  $A \cap B$  \_\_\_\_\_
3.  $S \cup T$  \_\_\_\_\_
4.  $S \cap T$  \_\_\_\_\_
5.  $M \cup N$  \_\_\_\_\_
6.  $M \cap N$  \_\_\_\_\_

V. Number of Subsets:

1. List all the subsets of  $\{0, 1, 2\}$   
\_\_\_\_\_
2. How many subsets are there in the set  $\{m, a, t, h\}$ ? \_\_\_\_\_
3. List all the subsets of  $\{m, a, t, h\}$ .  
\_\_\_\_\_  
\_\_\_\_\_
4. How many subsets are there in the set  $\{2, 4, 6, 8, 10, 12\}$ ? \_\_\_\_\_

VI. Closure

Given the set  $B = \{0, 1, 2\}$ , is the set  $B$  closed under the following operations?

1. \_\_\_\_\_ Addition
2. \_\_\_\_\_ Subtraction
3. \_\_\_\_\_ Multiplication
4. \_\_\_\_\_ Division

Given the set  $D = \{\text{integers}\}$ , is the set  $D$  closed under the following operations?

5. \_\_\_\_\_ Addition
6. \_\_\_\_\_ Subtraction
7. \_\_\_\_\_ Multiplication
8. \_\_\_\_\_ Division